

# Linear operating region of two-op-amp instrumentation amplifiers with gain stages

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## Introduction

The linear operating region of instrumentation amplifiers depends on numerous factors that include topology, supply voltage, common-mode voltage ( $V_{CM}$ ), output voltage ( $V_{OUT}$ ), gain ( $G$ ), and reference voltage ( $V_{REF}$ ). Operation outside or on the edge of this region is the most common issue with instrumentation amplifiers found in the TI E2E™ Community. Such operation yields forum posts that describe distorted output waveforms, incorrect device gain, or ‘stuck’ outputs. When such behaviors are observed, it is important to verify that the device is operating within the linear region.

The three primary topologies of instrumentation amplifiers that require discussion of their linear operating regions are: three-op-amp, two-op-amp, and two-op-amp with gain stage. The linear operating regions of the first two topologies are well documented in a three-part article series, blog post, and Analog Applications Journal article.<sup>[1-3]</sup>

This article analyzes the instrumentation-amplifier topology with two operational amplifiers (op amps) and a gain stage (GS), including its linear operating region as defined by the  $V_{CM}$  vs.  $V_{OUT}$  plot. Additionally, the internal node equations are derived and used to plot the swing limits of the input common-mode and output of each internal amplifier as a function of the common-mode voltage of the instrumentation amplifier.

## The $V_{CM}$ vs. $V_{OUT}$ plot

The  $V_{CM}$  vs.  $V_{OUT}$  plot of an instrumentation amplifier captures the common-mode and output-swing limitations of all internal op amps. A typical  $V_{CM}$  vs.  $V_{OUT}$  plot for a two-op-amp instrumentation amplifier with gain stage is shown in Figure 1. In order to create the plot shown in Figure 1, the device’s input pins were shorted together to ensure a differential input of 0 V. The common-mode voltage was then swept from 0 V to 5 V for different values of the reference voltage, hence the term “REF increasing” in the figure.

Notice, however, that this plot is actually  $V_{OUT}$  vs.  $V_{CM}$ , which is contrary to the other two instrumentation amplifier topologies. While there is no particular reason for this orientation, the plot still defines the linear operating region of the device. Note that the orientation of the axes also depends on the semiconductor manufacturer.

Operating outside of the boundaries results in non-linear operation of the device as shown in Figure 2.

Figure 1. A plot for the linear operating region of a two-op-amp instrumentation amplifier with gain stage

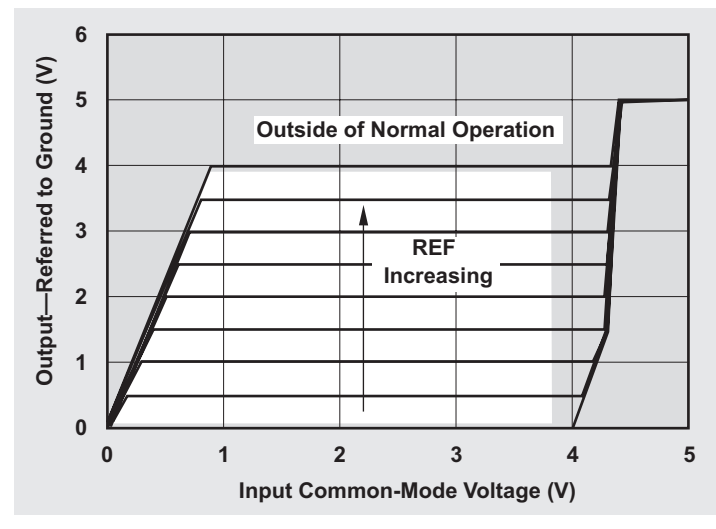
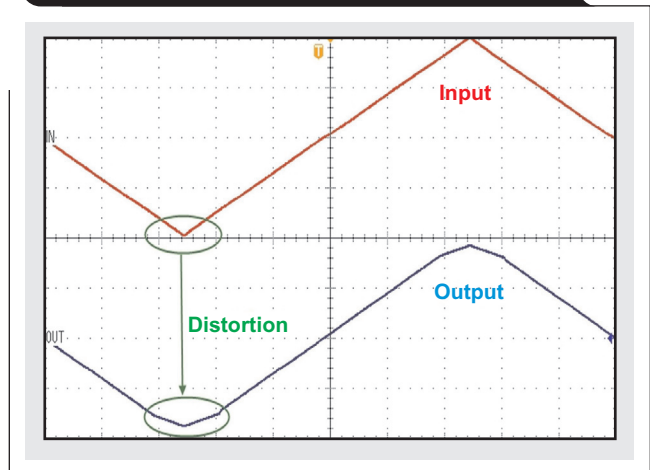


Figure 2. Instrumentation amplifier output distortion due to  $V_{CM}$  vs.  $V_{OUT}$  violation<sup>[1]</sup>



### Analysis of a two-op-amp instrumentation amplifier with gain stage

Figure 3 depicts the topology of a typical two-op-amp instrumentation amplifier with gain stage. This topology has high input impedance and requires two resistors,  $R_1$  and  $R_2$ , to set the gain.

One issue to consider is that the signal-path imbalance from  $V_{+IN}$  and  $V_{-IN}$  to the output can degrade the device's common-mode rejection ratio (CMRR) performance (Figure 4). In general, three-op-amp instrumentation amplifiers have a minimum DC CMRR of 100 dB, whereas the two-op-amp topologies have a DC CMRR of less than 100 dB. The degradation in CMRR is one of the primary reasons why the two-op-amp instrumentation amplifiers typically cost less than their three-op-amp counterparts.

Notice in Figure 4 that the CMRR curve does not change with gain, which is unlike the three-op-amp and the other two-op-amp instrumentation amplifiers. This is because CMRR is defined as the ratio of differential gain to common-mode gain. Since the differential gain of this instrumentation amplifier topology is fixed by the integrated resistors, CMRR does not change with gain.<sup>[4]</sup>

The transfer function for the topology shown in Figure 3 is given by Equation 1.

$$V_O = (V_{+IN} - V_{-IN}) \times G + V_{REF} = V_D \times G + V_{REF} \quad (1)$$

This transfer function is now derived to help understand the linear operating region of this topology. The first step is to determine the relationship between the integrated resistors ( $R_{FA1}$ ,  $R_{FA2}$ ,  $R_{OA1}$ , and  $R_R$ ) such that the gain applied to  $V_{REF}$  by the two-op-amp instrumentation amplifier is 1 V/V. To do this, a reference voltage is applied to the  $V_{REF}$  terminal and the  $V_{+IN}$  and  $V_{-IN}$  inputs are grounded (Figure 5).

Amplifier A1 applies an inverting gain to  $V_{REF}$  (Equation 2):

$$V_{OA1} = V_{REF} \times \left( \frac{-R_{FA1}}{R_R} \right) \quad (2)$$

Amplifier A2 applies an inverting gain to the output of amplifier A1 (Equation 3).

$$V_{OA2} = V_{OA1} \times \left( \frac{-R_{FA2}}{R_{OA1}} \right) \quad (3)$$

Substituting Equation 2 into Equation 3 yields Equation 4:

$$V_{OA2} = V_{REF} \times \left( \frac{-R_{FA1}}{R_R} \right) \times \left( \frac{-R_{FA2}}{R_{OA1}} \right) \quad (4)$$

Figure 3. Topology of a two-op-amp instrumentation amplifier with gain stage

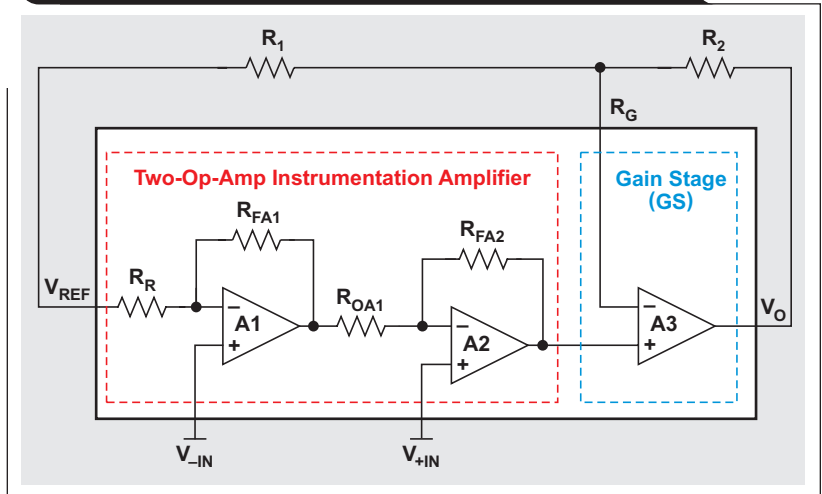


Figure 4. CMRR of a two-op-amp instrumentation amplifier with gain stage

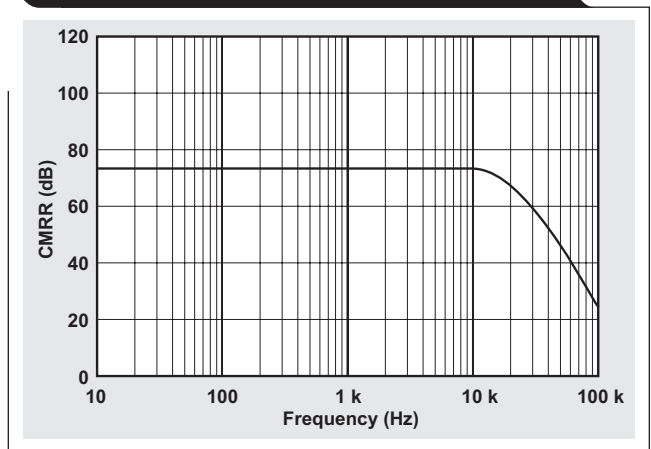
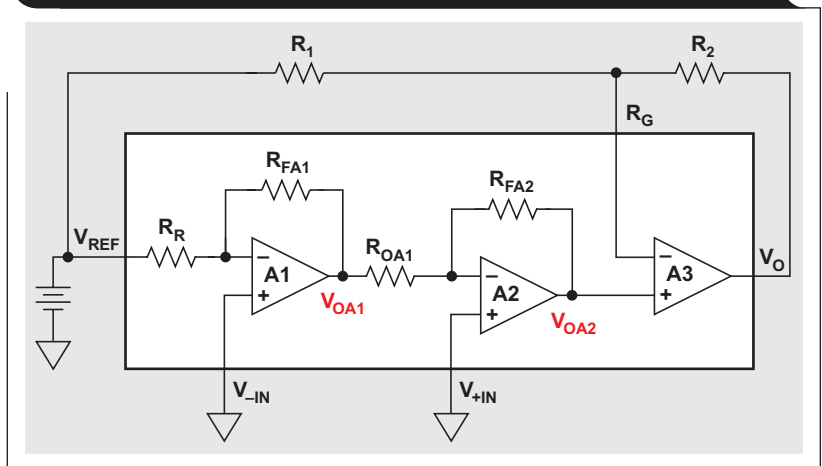


Figure 5. Apply reference voltage and ground input terminals



The gain applied by the two-op-amp instrumentation amplifier to the reference voltage should be 1 V/V. To fulfill this requirement, set  $R_{FA2} = R_R$  and  $R_{FA1} = R_{OA1} = R_F$ .

Figure 6 is a simplified version of Figure 5 to help show the effects of amplifier A3 on the reference voltage.

Amplifier A3 applies both an inverting (INV) and non-inverting (NI) gain to the reference voltage as given by Equations 5 and 6.

$$V_{O-NI} = V_{REF} \times \left(1 + \frac{R_2}{R_1}\right) \tag{5}$$

$$V_{O-INV} = V_{REF} \times \left(\frac{-R_2}{R_1}\right) \tag{6}$$

Equation 7 uses superposition to show that there is no gain applied to the reference voltage by amplifier A3.

$$\begin{aligned} V_O &= V_{O-NI} + V_{O-INV} \\ &= V_{REF} \times \left(1 + \frac{R_2}{R_1} + \frac{-R_2}{R_1}\right) = V_{REF} \end{aligned} \tag{7}$$

Figure 7 depicts the updated schematic that results in unity gain for the reference voltage. An input signal composed of a common-mode ( $V_{CM}$ ) and differential-mode ( $V_D$ ) voltage is added. Finally, all of the internal nodes are labeled for later analysis.

Each amplifier in Figure 7 inputs two signals; therefore, inverting gain and non-inverting gain applies. This yields six gain terms, as shown in Equations 8 through 13.

$$G_{A1\_INV} = \frac{-R_F}{R_R} \tag{8}$$

$$G_{A1\_NI} = \frac{R_R + R_F}{R_R} = 1 + \frac{R_F}{R_R} \tag{9}$$

$$G_{A2\_INV} = \frac{-R_R}{R_F} \tag{10}$$

$$G_{A2\_NI} = \frac{R_F + R_R}{R_F} = 1 + \frac{R_R}{R_F} \tag{11}$$

$$G_{A3\_INV} = \frac{-R_2}{R_1} \tag{12}$$

$$G_{A3\_NI} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1} \tag{13}$$

Figure 6. Simplified schematic to show effect of A3 on  $V_{REF}$

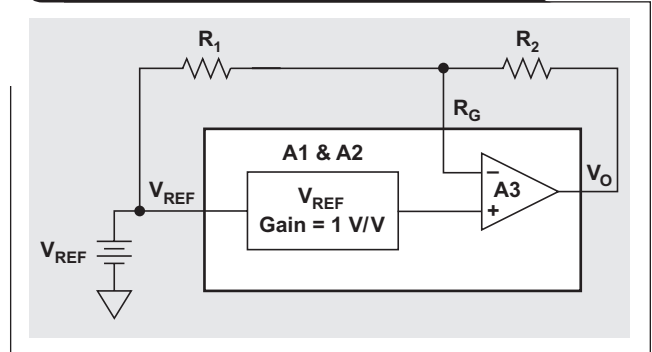
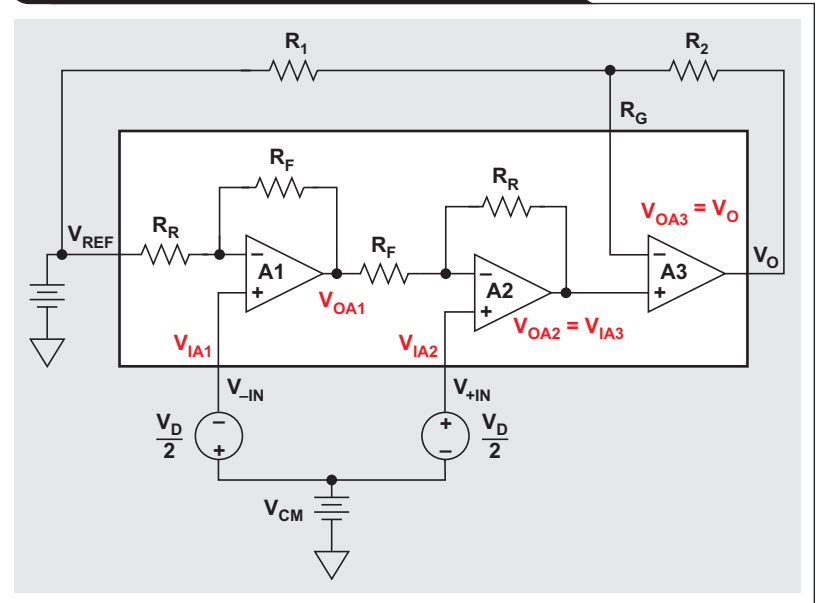


Figure 7. Internal nodes of the two-op-amp with gain-stage topology



Two important relationships between these gains are given by Equations 14 and 15.

$$G_{A1\_INV} \times G_{A2\_INV} = 1 \tag{14}$$

$$G_{A1\_NI} \times G_{A2\_INV} = -G_{A2\_NI} \tag{15}$$

Equations 16 through 18 define the output voltages of each amplifier.

$$V_{OA1} = V_{-IN} (G_{A1\_NI}) + V_{REF} (G_{A1\_INV}) \tag{16}$$

$$V_{OA2} = V_{+IN} (G_{A2\_NI}) + V_{OA1} (G_{A2\_INV}) \tag{17}$$

$$V_{OA3} = V_O = V_{OA2} (G_{A3\_NI}) + V_{REF} (G_{A3\_INV}) \tag{18}$$

Using Equations 14 through 18, the final transfer function for a two-op-amp instrumentation amplifier with gain stage is shown in Equation 19. It is consistent with Equation 1. Note that  $G = G_{A2\_NI} \times G_{A3\_NI}$ .

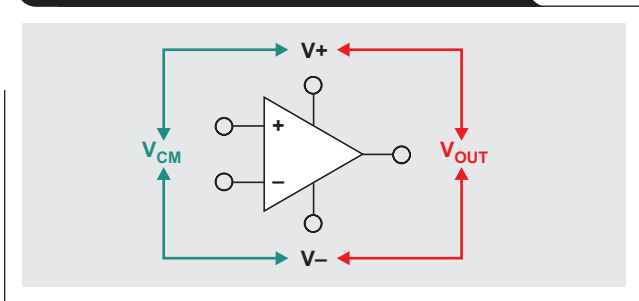
$$V_O = (V_{+IN} - V_{-IN}) \times (G_{A2\_NI} \times G_{A3\_NI}) + V_{REF} \tag{19}$$

$$= V_D \times \left(1 + \frac{R_R}{R_F}\right) \times \left(1 + \frac{R_2}{R_1}\right) + V_{REF}$$

### Op-amp limitations

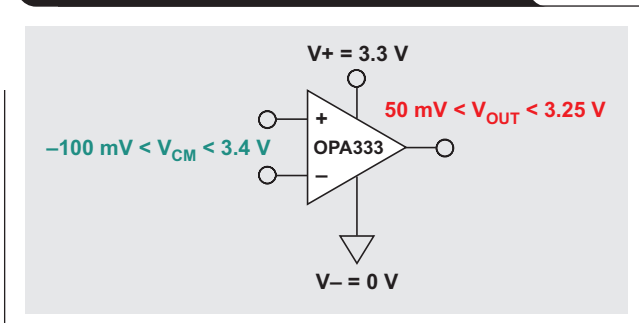
Linear operation of an instrumentation amplifier is contingent upon the linear operation of its primary building block: op amps. An op amp operates linearly when the input and output signals are within the device’s input common-mode and output-swing ranges, respectively. The supply voltages used to power the op amp ( $V_+$  and  $V_-$ ) define these ranges (Figure 8).

**Figure 8: Op-amp input common-mode ( $V_{CM}$ ) and output-swing ( $V_{OUT}$ ) ranges**



A real-world example of common-mode and output-swing limits is shown in Figure 9. Notice that the common-mode range and output-swing range are not necessarily the same.

**Figure 9. Op-amp  $V_{CM}$  and  $V_{OUT}$  ranges for a 3.3-V supply**



### Two-op-amp with gain-stage node equations

With a solid understanding of the two-op-amp instrumentation amplifier with gain stage and op-amp limitations, the next step is to examine the node equations shown in Figure 7. The equations for  $V_{OA1}$ ,  $V_{OA2}$ , and  $V_{OA3}$  are already given by Equations 16 through 18. Equations for  $V_{IA1}$  and  $V_{IA2}$  are given below.

$$V_{IA1} = V_{-IN} = V_{CM} - \frac{V_D}{2} \tag{20}$$

$$V_{IA2} = V_{+IN} = V_{CM} + \frac{V_D}{2} \tag{21}$$

The plot of the linear operating region can vary based on gain and reference voltage. Therefore, Equations 16 through 18 and 20 through 21 must be solved for  $V_O$  as a function of the gain terms,  $V_{CM}$ , and  $V_{REF}$ . A useful relationship is obtained by solving Equation 1 for  $V_D$ , as shown in Equation 22.

$$V_O = V_D \times G + V_{REF} \Rightarrow V_D = \frac{V_O - V_{REF}}{G} \tag{22}$$

After making all of the proper substitutions and solving for  $V_O$ , Equations 23 through 27 capture the linear operating region of a two-op-amp instrumentation amplifier with gain stage at the output ( $V_O$ ) as a function of the gain terms,  $V_{CM}$ ,  $V_{REF}$ , and the common-mode and output limitations of each amplifier ( $V_{IA1}$ ,  $V_{IA2}$ ,  $V_{IA3}$ ,  $V_{OA1}$ ,  $V_{OA2}$ ,  $V_{OA3}$ ).

$$V_{O\_IA1} = 2G \times (V_{CM} - V_{IA1}) + V_{REF} \tag{23}$$

$$V_{O\_IA2} = 2G \times (V_{IA2} - V_{CM}) + V_{REF} \tag{24}$$

$$V_{O\_OA1} = 2V_{CM}G + 2G_{A3\_NI} \times (V_{OA1}G_{A2\_INV} - V_{REF}) + V_{REF} \tag{25}$$

$$V_{O\_OA2} = V_{O\_IA3} = G_{A3\_NI} \times (V_{OA2} - V_{REF}) + V_{REF} \tag{26}$$

$$V_{O\_OA3} = V_{OA3} \tag{27}$$

In order to operate in a linear region, the voltage at  $V_{IA1}$  must not violate the input common-mode range of A1. Similarly, the voltage at node  $V_{OA1}$  must not violate the output swing limitation of A1. The same holds true for the common-mode and output-swing limitations of A2 and A3. The limitations of the internal amplifiers are usually obtained by inspecting the device’s data sheet and/or measuring the linear operating region in the lab.

Figure 10 depicts a TINA-TI™ simulation that plots Equations 23 through 27 for both the maximum and minimum common-mode and output-swing limits for the internal amplifiers of the INA331. The linear operating region is the interior of all lines.

The software tool introduced in Reference 3 was modified to include the ability to plot the linear operating region of two-op-amp instrumentation amplifier with gain stages (for example, INA321, INA322, INA331, and INA332). This simplifies the creation of the plots for varying gains, reference voltages, and supply voltages. See Related Web sites for a download link to the tool. Figure 11 depicts the plot for the INA331 given standard data sheet conditions. Notice that after rotating and mirroring the plot, it compares well with Figures 1 and 10. Finally, note that the software tool can be downloaded to generate the linear operating region of all three instrumentation amplifier topologies.

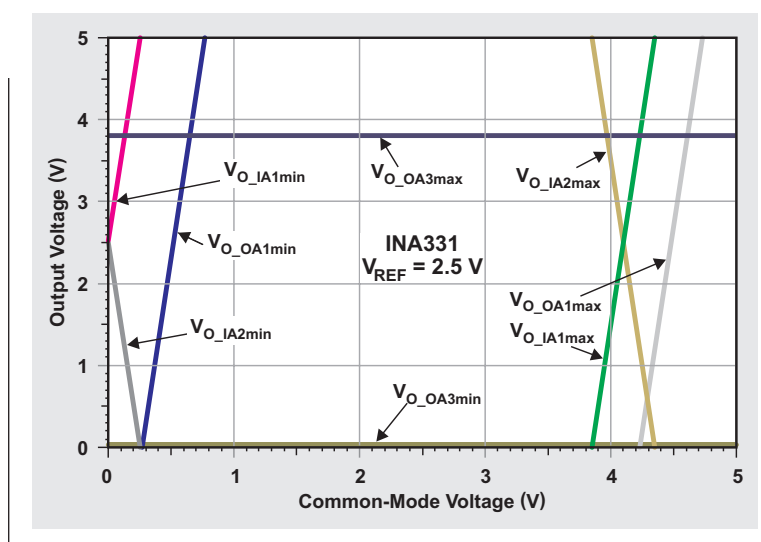
## Conclusions

The high frequency of questions on the TI E2E™ Community concerning the linear operating region of instrumentation amplifiers, often referred to as  $V_{CM}$  vs.  $V_{OUT}$  plots, shows that user interpretation is often misunderstood. The analysis of the two-op-amp instrumentation amplifier with gain stage set forth in this article, as well as References 1 through 4 below, can shorten the time required to locate problems with instrumentation amplifier designs. Furthermore, to simplify the task of ensuring linear operation of instrumentation amplifiers in future designs, download and install the free tool,  $V_{CM}$  vs.  $V_{OUT}$  plot generator.

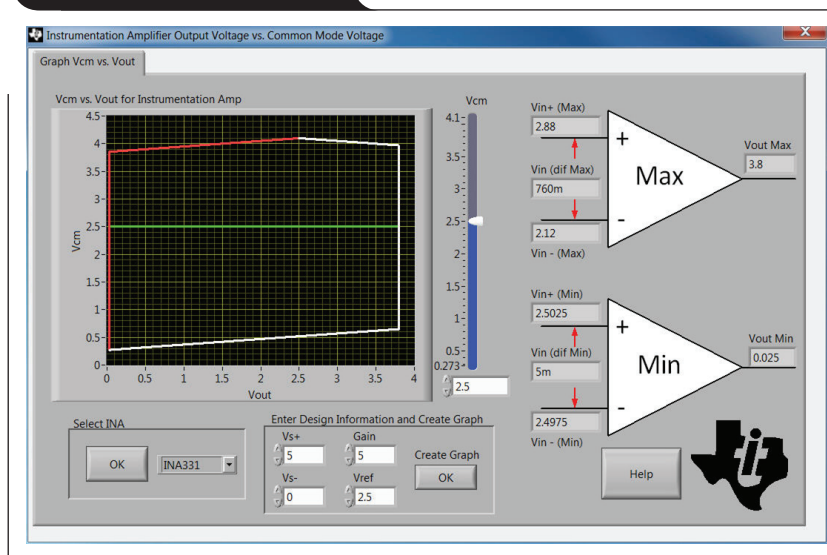
## References

1. Peter Semig and Collin Wells, "Instrumentation amplifier  $V_{CM}$  vs.  $V_{OUT}$  plots," Part 1, Part 2 and Part 3, EDN Network, December 2014
2. Peter Semig, "How Instrumentation Amplifier  $V_{CM}$  vs.  $V_{OUT}$  plots change with supply and reference voltage," TI Precision Hub blog, January 30, 2015
3. Peter Semig, " $V_{CM}$  vs.  $V_{OUT}$  plots for Instrumentation amplifiers with two op amps," Analog Applications Journal (SLYT647), 4Q 2015
4. Peter Semig, "Why doesn't my INA CMRR change with gain?" TI Precision Hub blog, February 28, 2014

**Figure 10. Simulated linear operating region of the INA331**



**Figure 11. Software tool**



## Related Web sites

Software tool:

**$V_{CM}$  vs.  $V_{OUT}$  plot generator**

Product information:

**INA331**

## Acknowledgements

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